

5-3

## Learn Check

In this Learning Check, you will be assessed on the following learning goals:

I can use the definition of derivative to compute derivatives

I can use derivatives and their graphs to identify properties of functions

- 1a. Let  $f$  be the function  $f(x) = 3x^2 - 2x + 4$ . Use the definition of derivative to find  $f'(x)$ .

$$\begin{aligned}
 f'(x) &= \lim_{\Delta x \rightarrow 0} \frac{3(x + \Delta x)^2 - 2(x + \Delta x) + 4 - (3x^2 - 2x + 4)}{\Delta x} \\
 &= \lim_{\Delta x \rightarrow 0} \frac{3(x^2 + 2x\Delta x + \Delta x^2) - 2x - 2\Delta x + 4 - 3x^2 + 2x - 4}{\Delta x} \\
 &= \lim_{\Delta x \rightarrow 0} \frac{\cancel{3x^2} + 6x\Delta x + 3\Delta x^2 - \cancel{2x} - 2\Delta x + \cancel{4} - \cancel{3x^2} + \cancel{2x} - \cancel{4}}{\Delta x} \\
 &= \lim_{\Delta x \rightarrow 0} \frac{6x + 3\Delta x - 2}{1} = 6x - 2
 \end{aligned}$$

- 1b Find the slope of the tangent line to  $f(x)$  at  $x = -8$ .

$$f'(-8) = 6(-8) - 2 = -48 - 2 = -50$$

- 1c. Is  $f$  increasing or decreasing when  $x = -8$ ? Justify your answer.

$f$  is decreasing. The slope of the tangent line at that point is negative.

3. A projectile follows along a path given by the formula  $h(t) = 480t - 16t^2$ . The derivative function for  $h(t)$  is  $h'(t) = 480 - 32t$  ( $t$  in seconds,  $h(t)$  in feet). Answer the below questions based on the functions.

- 3a. Find the instantaneous velocity when  $t = 20$ . Explain what your answer means about the projectile.

$$h'(20) = 480 - 32(20) = -160 \rightarrow \text{Height is decreasing}$$

The projectile is traveling down at 160 ft/s after 20 seconds.

- 3b. Find the instantaneous velocity when  $t = 8$ . Explain what your answer means about the projectile.

$$h'(8) = 480 - 32(8) = 224 \rightarrow \text{Height is increasing}$$

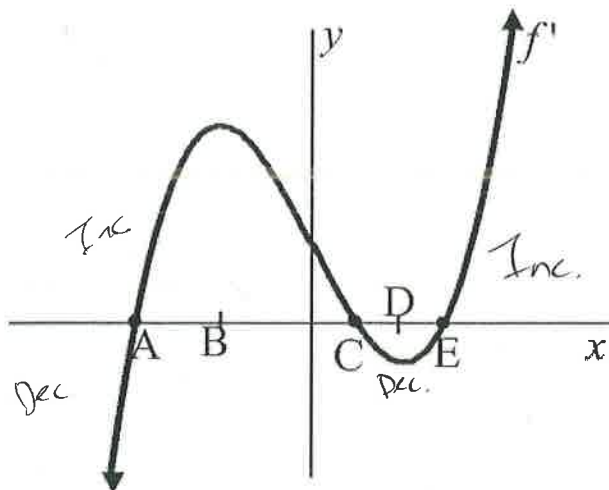
After 8 seconds, the projectile is traveling upward at a speed of 224 ft/sec.

- 3c. Find the instantaneous velocity when  $t = 15$ . Explain what your answer means about the projectile.

$$h'(15) = 480 - 32(15) = 0 \rightarrow \text{Projectile at maximum height}$$

After 15 seconds, the projectile is sitting still.

Below is a graph of  $f'(x)$  (the derivative function). Use the graph to answer the below questions. Remember, the derivative is the instantaneous rate of change at a specific  $x$ -value.



5. On what intervals (use A, B, C, D, and E) is the graph of  $f(x)$  increasing?

$(A, C)$   $(E, \infty)$   
Between A & C To the right of E

Derivative  
is  
positive

6. On what intervals (use A, B, C, D, and E) is the graph of  $f(x)$  decreasing?

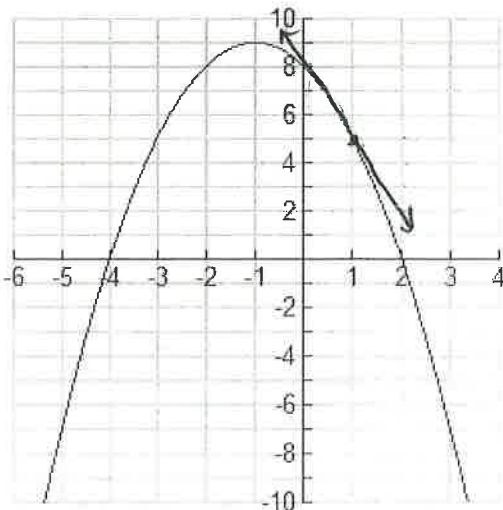
$(-\infty, A)$   $(C, E)$

Derivative  
is  
negative

7. What do the  $x$ -intercepts of the above graph of  $f'(x)$  tell you about the graph of  $f(x)$ ?

$f(x)$  has max or min points at those three  $x$ -values.  
Specifically, minimums at A & E; maximum at C.

Use the graph below to estimate the value of the derivative at the given points.



8.  $x = -1$

$$q'(-1) = 0$$

9.  $x = 1$

$$q'(1) = \frac{-3}{1}$$